Module

9

AC Theory

# LCR Series Circuits

## Introduction to LCR Series Circuits

### What you'll learn in Module 9.

### Module 9 Introduction

Introduction to LCR Series Circuits.

### Section 9.1 LCR Series Circuits.

Recognise LCR Series circuits and describe their action using phasor diagrams and appropriate equations:

Below Resonance
Above Resonance
At Resonance

### Section 9.2 Series resonance.

Describe LCR Series Circuits at resonance.
Describe the conditions for series resonance.
Carry out calculations on LCR series circuits, involving reactance, impedance, voltages and current.

### Section 9.3 Voltage Magnification.

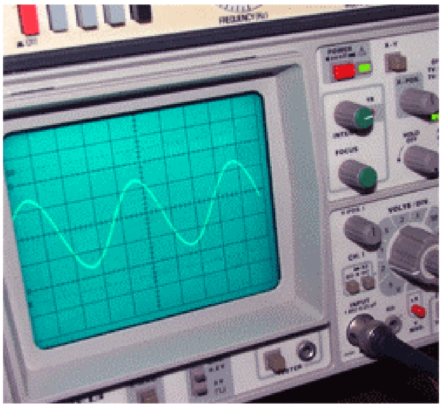
Describe voltage magnification in LCR Series Circuits
Calculate Voltage Magnification using appropriate formulae.

### Section 9.4 LCR Series Quiz.

LCR Series Circuits Quiz.

### Amazing LCR Circuits.

This module introduces some of the most useful and most amazing circuits in electronics. They can be as simple as two or three components connected in series, but in their operation they can perform many complex tasks and are used perhaps, in more circuit applications than any other circuit arrangement.



Connecting an inductor, a capacitor and perhaps a resistor, either in series or in parallel, makes some surprising things happen. Previous modules in this series have examined capacitors and inductors in isolation, and combined with resistors. These have created useful circuits such as filters, differentiators and integrators. Now module 9 looks at what happens when inductors and capacitors are combined in a single circuit network.

Capacitors and inductors act in different (and often opposite) ways in AC circuits. This module is about combining the properties of reactance and impedance of capacitors and inductors with varying frequency to produce amazing effects.

A circuit containing L,C and R at a certain frequency can make L and C (or at least their electrical effects) completely disappear! The LCR circuit can appear to be just a capacitor, just an inductor, or solely a resistor! Not only that, the series LCR circuit can magnify voltage, so the voltages across individual components within the circuit, can actually be much larger than the external voltage supplying the circuit. LCR circuits can also dramatically change their impedance to offer more or less opposition to current at different frequencies. All these effects can be used separately or together to make the wide range of electronic devices that use AC.

# Module 9.1 LCR Series Circuits.

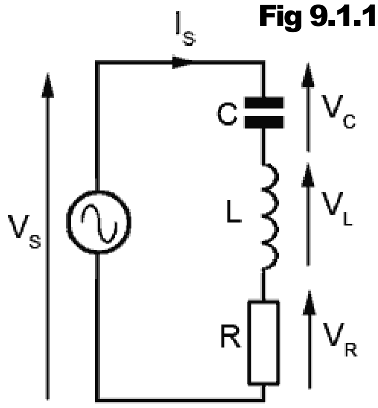



**Fig 9.1.1**

The circuit in Fig 9.1.1 contains all the elements so far considered separately in modules 1 to 8, namely inductance, capacitance and resistance, as well as their properties such as Reactance, Phase, Impedance etc.

This module considers the effects of L C and R connected together in series and supplied with an alternating voltage. In such an arrangement, the same circuit supply current ($I_S$) flows through all the components of the circuit, and $V_R$ $V_L$ and $V_C$ indicate the voltages across the resistor, the inductor and the capacitor respectively.

Module 6.1 described the effect of internal resistance on the voltage measured across an inductor. In LCR circuits both internal (inductor) resistance, and external resistance are present in the complete circuit. Therefore, it will be easier to begin with, to consider that the voltage $V_R$ is the voltage across the TOTAL circuit resistance, which comprises the internal resistance of L, added to any separate fixed resistor. Where $V_S$ is mentioned, this is the applied supply voltage.

The phase relationship between the supply voltage $V_S$ and the circuit current $I_S$ depends on the frequency of the supply voltage, and on the relative values of inductance and capacitance, and whether the inductive reactance ($X_L$) is greater or less than the capacitive reactance ($X_C$). There are various conditions possible, which can be illustrated using phasor diagrams.

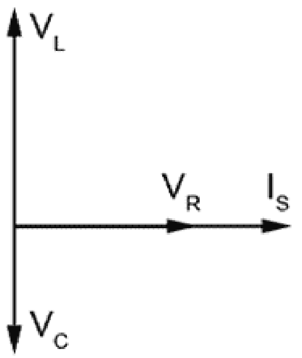



**Fig 9.1.2 Phasors for $V_L$ and $V_C$ are in anti phase.**

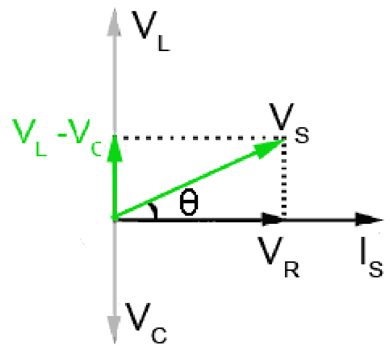



**Fig 9.1.3 $V_L$ is greater than $V_C$ so the circuit behaves like an inductor**

Fig 9.1.2 shows the circuit conditions when the inductive reactance ($X_L$) is greater than the capacitive reactance ($X_C$). In this case, since both L and C carry the same current, and $X_L$ is greater than $X_C$, it follows that $V_L$ must be greater than $V_C$.

**$V_L = I_S X_L$ and $V_C = I_S X_C$**

Remember that $V_C$ and $V_L$ are in anti-phase to each other due to their 90° leading and lagging relationship with the circuit current ($I_S$). As $V_L$ and $V_C$ directly oppose each other, a resulting voltage is created, which will be the difference between $V_C$ and $V_L$. This is called the REACTIVE VOLTAGE and its value can be calculated by simply subtracting $V_C$ from $V_L$. This is shown in Fig 9.1.3 by the phasor ($V_L - V_C$).

The length of the phasor ($V_L - V_C$) can be arrived at graphically by removing a portion from the tip of the phasor ($V_L$), equivalent to the length of phasor ($V_C$).

$V_S$ is therefore the phasor sum of the reactive voltage ($V_L - V_C$) and $V_R$. The phase angle θ shows that the circuit current $I_S$ lags on the supply voltage $V_S$ by between 90° and 0°, depending on the relative sizes of ($V_L - V_C$) and $V_R$. Because $I_S$ lags $V_S$, this must mean that the circuit is mainly inductive, but the value of inductance has been reduced by the presence of C. Also the phase difference between $I_S$ and $V_S$ is no longer 90° as it would be if the circuit consisted of only pure inductance and resistance.

Because the phasors for ($V_L - V_C$), $V_R$ and $V_S$ in Fig 9.1.3 form a right angle triangle, a number of properties and values in the circuit can be calculated using either Pythagoras´ Theorem or some basic trigonometry, as illustrated in "Using Phasor Diagrams" in Module 5.4.

For example:

$$V_S^2 = (V_L - V_C)^2 + V_R^2 \quad \text{therefore} \quad V_S = \sqrt{(V_L - V_C)^2 + V_R^2}$$

The total circuit impedance (Z) can be found in a similar way: The phase angle between ($V_L - V_C$) and

$$Z = \sqrt{(X_L - X_C)^2 + R^2}$$

$V_R$ can be found using trigonometry as illustrated in "Using Phasor Diagrams" in Module 5.4.

**tan θ = opposite ÷ adjacent**, therefore **tan θ = (VL − VC) ÷ VR**

so to find the angle θ

$$\theta = \tan^{-1} \frac{(V_L - V_C)}{V_R}$$

Also, Ohms Law states that R (or X) = V / I

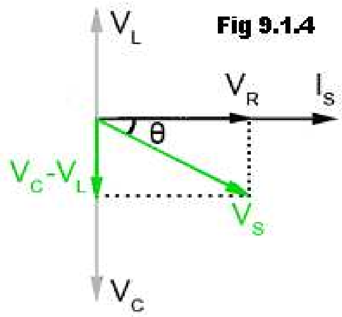
Therefore if ($V_L - V_C$) and $V_R$ are each divided by the current ($I_S$) this allows the phase angle θ to be found using the resistances and reactances, without first working out the individual voltages.

$$\theta = \tan^{-1} \frac{(X_L - X_C)}{R}$$

This can be useful when component values need to be chosen for a series circuit, to give a required angle of phase shift.

### When VC is larger than VL the circuit is capacitive.

Fig 9.1.4 illustrates the phasor diagram for a LCR series circuit in which $X_C$ is greater than $X_L$ showing that when $V_C$ exceeds $V_L$ the situation illustrated in Fig 9.1.3 is reversed.




The resultant reactive voltage is now given by ($V_C - V_L$) and $V_S$ is the phasor sum of ($V_C - V_L$) and $V_R$.

The phase angle θ now shows that the circuit current ($I_S$) leads supply voltage ($V_S$) by between 0° and 90°. The overall circuit is now capacitive, but less so than if L was not present.

In using the above formulae, remember that the reactive value (the difference between $V_L$ and $V_C$ or $X_L$ and $X_C$) is given by subtracting the smaller value from the larger value. For example, when $V_C$ is larger than $V_L$:

$$V_S = \sqrt{(V_C - V_L)^2 + V_R^2}$$

Looking at the phasor diagrams for a LCR series circuit it can be seen that the supply voltage ($V_S$) can either lead or lag the supply current($I_S$) depending largely on the relative values of the component reactances, $X_L$ and $X_C$.

### When VL and VC are equal the circuit is purely resistive.

As shown in Module 6.1 and 6.2, the reactance of L and C depends on frequency, so if the frequency of the supply voltage $V_S$ is varied over a suitable range, the series LCR circuit can be made to act as either an inductor, or as a capacitor, but that's not all.

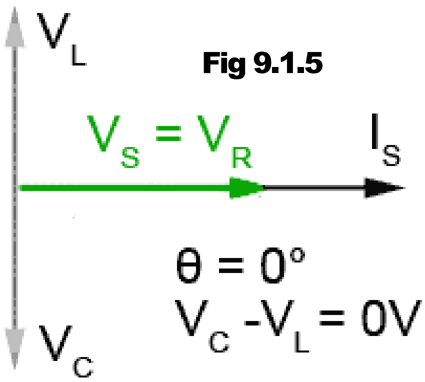



Fig 9.1.5 shows the situation, which must occur at some particular frequency, when $X_C$ and $X_L$ (and therefore $V_C$ and $V_L$) are equal.

The opposing and equal voltages $V_C$ and $V_L$ now completely cancel each other out. **The supply voltage and the circuit current must now be in phase, so the circuit is apparently entirely resistive!** L and C have completely "disappeared".

This special case is called SERIES RESONANCE and is explained further in Module 9.2.

# Module 9.2 Series Resonance

**Series Resonance happens when reactances are equal.**

Inductive reactance ($X_L$) in terms of frequency and inductance is given by:

$$X_L = 2\pi f L$$

and capacitive reactance ($X_C$) is given by:

$$X_C = \frac{1}{2\pi f C}$$

Inductive reactance is **directly proportional** to frequency, and its graph, plotted against frequency ($f$) is a straight line.

Capacitive reactance is **inversely proportional** to frequency, and its graph, plotted against $f$ is a curve. These two quantities are shown, together with R, plotted against $f$ in Fig 9.2.1 It can be seen from this diagram that where $X_C$ and $X_L$ intersect, they are equal and so a graph of ($X_L - X_C$ ) must be zero at this point on the frequency axis.

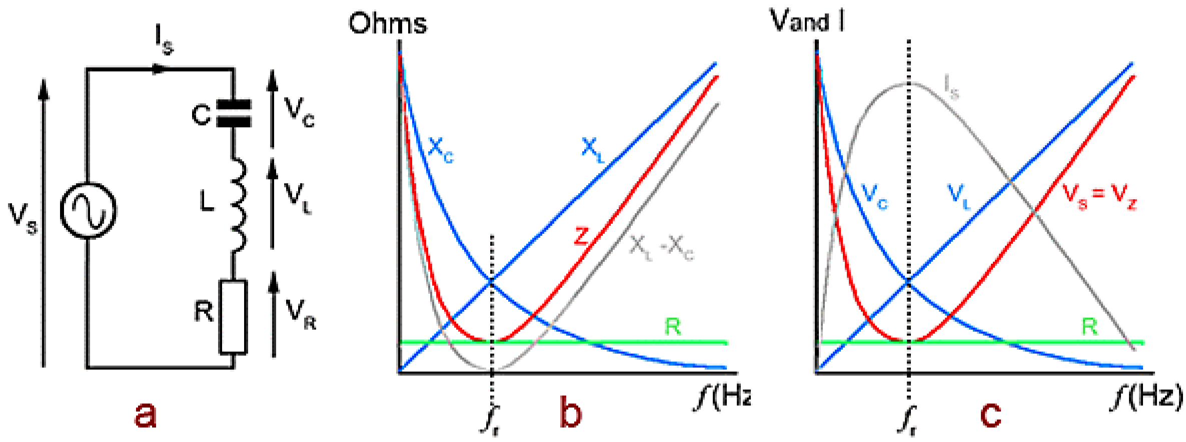



**Fig 9.2.1 The Properties of a Series LCR Circuit at Resonance.**

Fig 9.2.1a shows a series LCR circuit and Fig 9.2.1b shows what happens to the reactances ($X_C$ and $X_L$), resistance (R) and impedance (Z) as the supply ($V_S$) is varied in frequency from 0Hz upwards. At first the circuit behaves as a capacitor, the total impedance of the circuit (Z) falls in a very similar curve to $X_L - X_C$.

Fig 9.2.1c illustrates the relationships between the individual component voltages, the circuit impedance(Z) and the supply current ($I_S$) (which is common to all the series components).

At a particular frequency $f_r$ it can be seen that $X_L - X_C$ has fallen to zero and only the circuit resistance R is left across the supply. The current flowing through the circuit at this point will therefore be at a maximum. Now $V_C$ and $V_L$ are equal in value and opposite in phase, so will completely cancel each other out. Reactance is effectively zero and the circuit is completely resistive, with Z equal to R. The circuit current ($I_S$) will be at its maximum and will be in phase with the supply voltage ($V_S$) which is at its minimum.

As the frequency increases above this resonant frequency ($f_r$) the impedance rises, and as $X_L$ is now the larger of the two reactances, the impedance curve begins to follow an increasing value more like the linear graph of $X_L$.

At frequencies below resonance the circuit behaves like a capacitor, at resonance as a resistor, and above $f_r$ the circuit behaves more and more like an inductor, and the graph of $X_L - X_C$ soon becomes an almost straight line.

This behaviour of a LCR Series Circuit allows for the statement of a number of useful facts about a series circuit that relate to its resonant frequency $f_r$.

### 6 Things you need to know about LCR Series Circuits.

1. **AT RESONANCE ($f_r$)** $V_C$ is equal to, but in anti-phase to $V_L$
2. ; **AT RESONANCE ($f_r$)** Impedance (Z) is at minimum and equal to the RESISTANCE (R)
3. **AT RESONANCE ($f_r$)** Circuit current ($I_S$) is at a maximum.
4. **AT RESONANCE ($f_r$)** The circuit is entirely resistive.
5. **BELOW RESONANCE ($f_r$)** The circuit is capacitive.
6. **ABOVE RESONANCE ($f_r$)** The circuit is inductive.

### Formula for Series Resonance.

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

The fact that resonance occurs when $X_L = X_C$ allows a formula to be constructed that allows calculation of the resonant frequency ($f_r$) of a circuit from just the values of L and C. The most commonly used formula for the series LCR circuit resonant frequency is:

### Where does the formula for $f_r$ come from?

The formula for finding the resonant frequency can be built from the two basic formulae that relate inductive and capacitive reactance to frequency.

$$X_L = 2\pi f L \quad \text{and} \quad X_C = \frac{1}{2\pi f_r C}$$

At the resonant frequency $f_r$ of an LC circuit, the values of $X_L$ and $X_C$ are equal, so their formulae must also be equal.

$$X_L = 2\pi f L = X_C = \frac{1}{2\pi f_r C}$$

Multiplying both sides of the equation by $2\pi f_r C$ removes the fraction on the right and leaves just a single term of $f$ (in the term 4 π 2*f*r2 LC) on the left.

$$4\pi^2 f_r^2 LC = 1$$

Dividing both sides of the result by $4\pi^2$ LC leaves just $f_r^2$ on the left.

$$f_r^2 = \frac{1}{(4\pi^2 LC)}$$

Finally, taking the square root of both sides gives an equation for $f_r$ and a useful formula for finding the resonant frequency of an LC circuit.

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

### Series Circuit Calculations.

In a series LCR circuit, especially at resonance, there is a lot happening, and consequently calculations are often multi stage. Formulae for many common calculations have been described in earlier modules in this series. The difference now is that the task of finding out relevant information about circuit conditions relies on selecting appropriate formulae and using them in a suitable sequence.

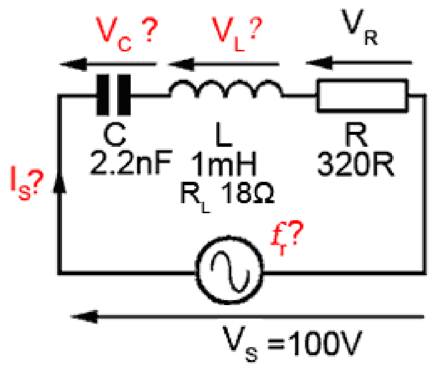
For example, in the problem below, values shown in red on the circuit diagram are required, but notice that $V_C$ and $V_L$ can't be worked out first, as a value for $f_r$ (and another formula) is needed to calculate the reactance. Sometimes the task is made easier by remembering the 6 useful facts (page 6) about series resonance. In example 9.2.2 below there is no need to calculate both $V_C$ and $V_L$ because, at resonance $X_C$ and $X_L$ are equal, so calculate one and you know the other!

Notice however that $V_L$ is not the same as the total voltage measured across L. The voltage across the internal resistance (at 90°to $V_L$) needs to be included, and because of the phase difference between $V_L$ and the internal resistance voltage ($V\,R_L$), the total measurable inductor voltage $V_{L\,TOT}$ will be the phasor sum of $V_L$ and $V\,R_L$.

### Example 9.2.2 Series LCR Circuit Calculations.

For a series LCR circuit comprising:
L (1mH, internal resistance 18Ω,) C (2.2nF) and R (320Ω) connected to a 100V AC supply:
Calculate the resonant frequency and the maximum current.
Calculate the voltage $V_C$ across C and the measurable voltage $V_{L\,TOT}$ across L.

1. Draw the circuit and list known values.
   L = 1mH
   $R_L$ (internal R = 18Ω)
   C = 2.2nF
   R = 320Ω
   $V_S$ = 100V




2. Decide on suitable formulae for unkown values.

(i) $f_r = \dfrac{1}{2\pi\sqrt{(LC)}}$

(ii) $I_S$ (at $f_r$) $= \dfrac{V}{Z}$ or $\dfrac{V}{R_{TOT}}$ where $R_{TOT} = R + R_L$

(iii) $V_C = IX_C$ (and $X_C = \dfrac{1}{2\pi f_r C}$)

(iv) $V_L$ $V_L = V_C$ at resonance $f_r$

(v) Total voltage across L ($V_{L\,TOT}$) $V_{L\,TOT} = \sqrt{(V_L^2 + V_{R_L}^2)}$

Work out each of these formulae (with pencil and paper and a calculator) remembering to work out the bracketed parts of the formula first, then check your answers by reading the text in  in Module 9.3

Working this way while learning, is a good way to help understand how the maths work. There are of course a good many LCR calculators on the web but take a tip, WORK IT OUT FIRST, then try a web calculator (or more than one, as some are cleverer than others) to check your answer.

# Module 9.3 Voltage Magnification

In the **answers** to the calculations in example 9.2.2 it should be noticeable that, at the circuit´s **resonant frequency $f_r$ of 107kHz,** the reactive voltages across L and C are equal and each is greater than the circuit supply voltage $V_S$ of 100V.

This is possible because, at resonance the **voltage ($V_C$ = 199.56V)** across the capacitor, is in anti−phase to the **voltage ($V_L$ = 199.56V)** across the inductance. As these two voltages are equal and opposite in phase, they completely cancel each other out, leaving only the supply voltage developed across the circuit impedance, which at resonance is the same as the total resistance of 320 + 18 = 338Ω.

At the resonant frequency the **current through the circuit is at a maximum value of about 296mA.** Because of the anti phase cancelling effect at resonance, the two reactive voltages $V_C$ and $V_L$ have "disappeared"! This leaves the supply current $I_S$ effectively flowing through R and the inductor resistance $R_L$ in series.

In this example the effect of the inductor´s 18Ω internal resistance on $V_L$ is so small (0.03V) as to be negligible and **$V_{L\ TOT}$ is the same value as $V_L$ at approximately 199.6V.**.

As the total circuit impedance is less than either the capacitive or inductive reactances at resonance, the supply voltage of 100V (developed across the circuit resistance) is less than either of the opposing reactive voltages $V_C$ or $V_L$. This effect, where the internal component reactive voltages are greater than the supply voltage is called VOLTAGE MAGNIFICATION.

This can be a very useful property, and is used for example in the antenna stages of radio receivers where a series circuit, resonant at the frequency of the transmission being received, is used to magnify the voltage amplitude of the received signal voltage, before it is fed to any transistor amplifiers in the circuit.

The voltage magnification that takes place at resonance is given the symbol Q and the "Q Factor" (the voltage magnification) of LC Band Pass and Band Stop filter circuits for example, controls the "rejection", the ratio of the wanted to the unwanted frequencies that can be achieved by the circuit.

The effects of voltage magnification are particularly useful as they can provide magnification of AC signal voltages using only passive components, i.e. without the need for any external power supply.

In some cases voltage magnification can also be a dangerous property. in high voltage mains (line) operated equipment containing inductance and capacitance, care must be taken during design to ensure that the circuit does not resonate at frequencies too close to that of the mains (line) supply. If that should happen, extremely high reactive voltages could be generated within the equipment, with disastrous consequences for the circuit and / or the user.

The Q factor can be calculated using a simple formula. The ratio of the supply voltage $V_S$ to either of the (equal) reactive voltages $V_C$ or $V_L$ will be in the same ratio as the total circuit resistance (R) is to either of the reactances ($X_C$ or $X_L$) at resonance. The ratio of the reactive voltage $V_L$ to the supply voltage $V_S$ is the magnification factor Q.

The formula for finding Q (the voltage magnification) uses the ratio of the inductive reactance to the total circuit resistance.

Where $X_L$ is the inductive reactance **at resonance**, given by $2\pi f_r L$ and R is the TOTAL circuit resistance. Note that Q does not have any units (volts, ohms etc.), as it is a RATIO

$$Q = \frac{X_L}{R} \text{ or } \frac{2\pi f_r L}{R}$$

**Question:** What is the magnification factor Q of the circuit in Example 9.2.2 in Module 9.2?

(No answer given, this one is down to YOU!)

# Module 9.4 LCR Series Quiz

## What you should know.

**After studying Module 9, you should:**

*Be able to recognise LCR Series circuits and describe their action using phasor diagrams and appropriate equations.*

*Be able to describe LCR Series Circuits at resonance and the conditions for series resonance.*

*Be able to carry out calculations on LCR series circuits, involving reactance, impedance, component and circuit voltages and current.*

*Be able to Describe voltage magnification and calculate Q factor in LCR Series Circuits*

Try our quiz, based on the information you can find in Module 9. Submit your answers and see how many you get right, but don't be disappointed if you get answers wrong. Just follow the hints to find the right answer and learn more about LCR Series Circuits and Resonance as you go.

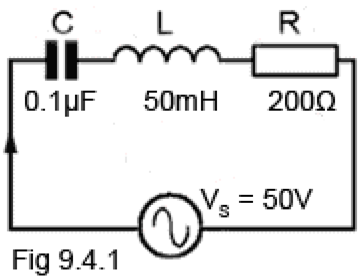


Fig 9.4.1

**1.**

With reference to Fig 9.4.1 the resonant frequency of the circuit will be approximately:

a) 71.2kHz

b) 444.3MHz

c) 2.251 kHz

d) 7.12MHz

**2.**

With reference to Fig 9.4.1, what will be the maximum supply current?

a) 70mA b) 250mA c) 500mA d) 14.14mA

**3.**

With reference to Fig 9.4.1, what will be the approximate voltage across C at resonance?

a) 177V b) 70V c) 1.7kV d) 353V

**4.**

With reference to Fig 9.4.1, what is the Q factor of the circuit?

a) 3.535V b) 1.4 c) 0.707 d) 3.5

Continued

**5.**

Which of the following statements about a series LCR circuit is true?

a) At resonance, the total reactance and total resistance are equal.

b) The impedance at resonance is purely inductive.

c) The current flowing in the circuit at resonance is at maximum.

d) The impedance at resonance is at maximum.

**6.**

If the values of L and C in a series LCR circuit are doubled, what will be the effect on the resonant frequency?

a) It will be halved.

b) It will not be changed.

c) It will double.

d) It will increase by four times.

**7.**

With reference to Fig 9.4.2, which phasor diagram shows a series LCR circuit at resonance?

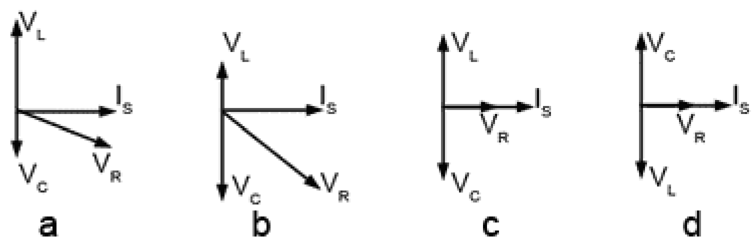



**Fig 9.4.2**

**8.**

What words are missing from the following statement? The impedance of series LCR circuit at resonance will be _______ and equal to the circuit _______ .

a) Minimum and resistance.

b) Maximum and resistance.

c) Minimum and reactance.

d) Maximum and reactance.

Continued

**9.**

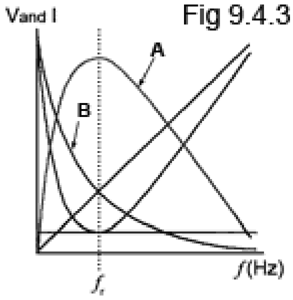
With reference to the graph of voltages and current in a series resonant circuit shown in Fig 9.4.3, What quantity is represented by line A?

a) Circuit impedance.

b) Voltage across the capacitor.

c) Supply voltage.

d) Circuit current.




**10.**

With reference to the graph of voltages and current in a series resonant circuit shown in Fig 9.4.3, What quantity is represented by line B?

a) Circuit impedance.

b) Voltage across the capacitor.

c) Supply voltage.

d) Circuit current.