Module
10

AC Theory

LCR Parallel Circuits

Introduction to LCR Parallel Circuits

What you'll learn in Module 10.

Module 10.1 Ideal Parallel Circuits.

Recognise ideal LCR parallel circuits and describe the effects of internal resistance.

Module 10.2 Practical Parallel Circuits.

Describe the action of practical LCR parallel circuits with the use of phasor diagrams.

Module 10.3 Parallel Resonance.

Describe the action of LCR parallel circuits above, below and at resonance.

Describe current magnification, and dynamic resistance in LCR parallel circuits.

Use appropriate formulae to carry out calculations on LCR parallel circuits, involving resonance, impedance and dynamic resistance.

Module 10.4 Damping.

Describe methods of damping in LCR parallel circuits, and relate Q factor and bandwidth.

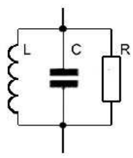
Module 10.5 Parallel Circuits Quiz.

LCR Parallel Circuits Quiz.

The LCR Parallel Circuit.

In introducing the LCR Series Circuit, one of the most useful combinations of "passive components" in electronics, Module 9 set the groundwork for Module 10. If the LCR series circuit is just one of the most useful circuits, here is the other one, the **LCR Parallel Circuit!**

The parallel LCR circuit uses the same components as the series version, its resonant frequency can be calculated in the same way, with the same formula, but just changing the arrangement of the three components from a series to a parallel connection creates some amazing transformations. Almost everything about the series circuit is turned upside down by the parallel circuit. As you read through this module, notice how many opposites there are between series and parallel circuits. It is because of these opposite effects, that series and parallel resonant circuits can together perform very many more important tasks in analogue electronics.



Module 10.1 The Ideal Parallel LCR Circuit.

The circuit in Fig 10.1.1 is an "Ideal" LC circuit consisting of only an inductor L and a capacitor C connected in parallel. Ideal circuits exist in theory only of course, but their use makes understanding of basic concepts (hopefully) easier. It allows consideration of the effects of L and C, ignoring any circuit resistance that would be present in a practical circuit.

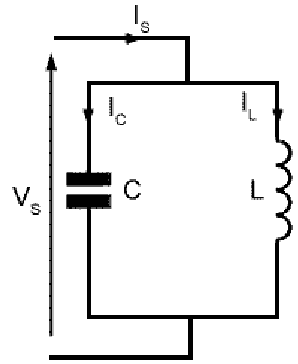
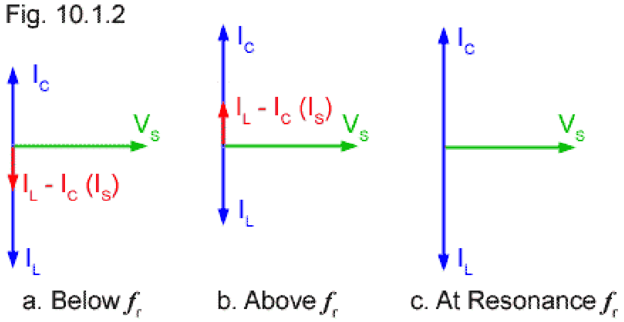


Fig 10.1.1

Fig 10.1.2 shows phasor diagrams for the circuit in Fig 10.1.1 under three different conditions, below, above and at resonance. Unlike the phasor diagrams for series circuits, these diagrams have a voltage V_S as the reference (horizontal) phasor, and have several phasors depicting currents. This is because, in a parallel circuit the voltage V_S is common to both the L and C arms of the circuit but each of the component arms (L and C) has individual CURRENTS.

Fig. 10.1.2



The phasors for L and C seem to be reversed compared with the phasor diagrams for series circuits in module 9, but the parallel phasor diagram shows the current I_C through the capacitor leading the supply voltage V_S by 90°, while the inductive current I_L lags the supply voltage by 90°. (The mnemonic CIVIL introduced in Module 5.1 still works for these diagrams.)

The supply current I_S will be the phasor sum of I_C and I_L but as, in the ideal circuit, there is no resistance present, I_C and I_L are in antiphase, and I_S will be simply the difference between them.

Fig 10.1.2a shows the circuit operating at some frequency below resonance f_r where I_L is greater than I_C and the total current through the circuit I_S is given by $I_L - I_C$ and will be in phase with I_L, and it will be lagging the supply voltage by 90°. Therefore at fre quencies below f_r more current flows through L than through C and so the parallel circuit acts as an INDUCTOR.

Fig 10.1.2b shows the conditions when the circuit is operating above f_r. Here, because X_C will be lower than X_L more current will flow through C. I_C is therefore greater than I_L and as a result, the total circuit current I_S can be given as $I_L - I_C$ but this time I_S is in phase with I_C. The circuit is now acting as a CAPACITOR.

> Notice that in both of the above cases the parallel circuit seems to act in the opposite manner to the series circuit described in Module 9. The series circuit behaved like a capacitor below resonance and an inductor above. The parallel circuit is acting like an inductor below resonance and a capacitor above. This change is because the parallel circuit action is considered in terms of current through the reactances, instead of voltage across the reactances as in the series circuit.

At resonance (f_r) shown in Fig 10.1.2c, the reactances of C and L will be equal, so an equal amount of current flows in each arm of the circuit, ($I_C = I_L$). This produces a very strange condition. Considerable current is flowing in each arm of the circuit, but the supply current is ZERO! There is no phasor for I_S! This impossible state of affairs of having currents flowing around the circuit with no supply current, indicates that the circuit must have infinite impedance to the supply. As there is no resistance in either L or C in the ideal circuit, current continues to flow from L to C and back again. This only happens of course in an ideal circuit, due to the complete absence of resistance in either arm of the circuit, but it is surprisingly close to what actually happens in a practical circuit, because current is in effect "stored" within the parallel circuit at resonance, without being released to the outside world. For this reason the circuit is sometimes also called a "tank circuit".

Module 10.2 Practical Parallel Circuits

Fig 10.2.1a Looking at the inductive (LR) branch of the parallel circuit.

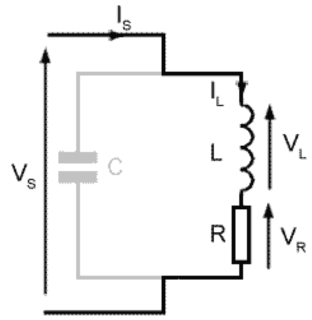


Fig 10.2.1a shows a practical LCR parallel circuit, where R is the internal resistance of the inductor L, plus any additional resistance in the inductive arm of the circuit. Before considering the whole circuit, the inductive branch will be examined as though it was a separate LR series circuit, and the arm containing C will be temporarily ignored. An understanding of what happens in L and R will be the foundation for a better understanding of the whole circuit.

Fig 10.2.1b Phasors for the L and R

Fig 10.2.1b shows a phasor diagram for the LR branch of the circuit in Fig 10.2.1a, drawn as it would be for an LR series circuit. The branch of the circuit containing C is being ignored. The reference phasor is (I_S) and because the same current (I_S) passes through both R and L, the phasors for I_L and V_R will be in the same phase. V_S is the phasor sum of V_L and V_R.
In a parallel circuit it will be the supply voltage V_S that is common to all components and so will be used as the reference phasor in Fig 10.2.2.

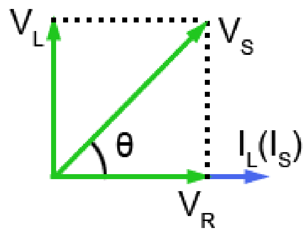
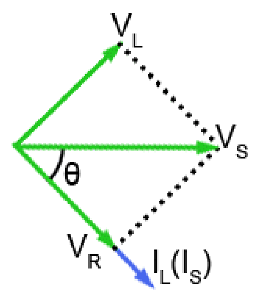


Fig 10.2.2 Phasors for the LR branch of a parallel LCR circuit

Fig 10.2.2 shows Fig 10.2.1b modified for a parallel circuit. The complete diagram is rotated so that the phasor for V_S is horizontal and used as the reference phasor. This is because, when describing PARALLEL circuits, it is the supply voltage (V_S) that is common to all components.

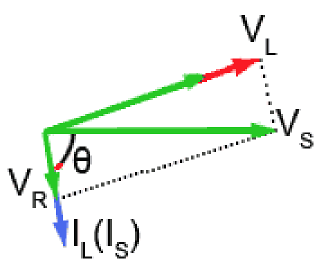


The phasors for I_L and V_R are in phase with each other, and V_L leads I_L by 90°. However the phase angle θ between V_S and I_L (and I_S) will vary with frequency. This is because the value of X_L and therefore V_L will increase as frequency increases. Because V_L changes in length, and V_S is fixed, angle θ will change, which will have an effect on the phasor diagrams for the complete LCR circuit.

Fig 10.2.3a Phasors for the LR branch of a parallel LCR circuit at HIGH frequency.

Fig 10.2.3a represents the condition when the frequency of the supply is high, so X_L and therefore V_L will be large. V_S is the phasor sum of V_R and V_L.

It follows then, that the phase angle θ is some value between 0° and 90° with I_L lagging on V_S. In the ideal circuit I_L always lags on V_S by 90°, so the effect of adding some resistance will be to reduce the angle of lag (θ). At higher frequencies however V_L and θ increase and the circuit becomes more like a pure inductor.

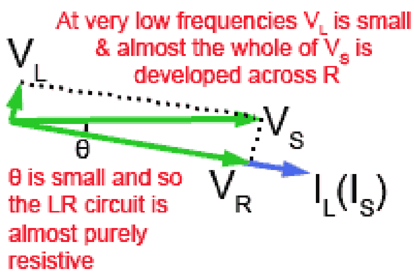


It is important to note that the value of X_L depends on both the frequency and the value of inductance. The value of R will also depend on the design of the inductor and so V_L and θ will depend on both the frequency of V_S and on component values.

Fig 10.2.3b Phasors for the LR branch of a parallel LCR circuit at LOW frequency.

Fig 10.2.3b shows the effect of reducing the frequency of V_S to a low value. X_L will now be smaller, and so will V_L.

V_S is still the phasor sum of V_R and V_L, due to the reduction in X_L, I_L will increase and most of the supply voltage will be developed across R, increasing V_R. With V_L reduced in amplitude and V_R increased, angle θ is very small making I_S and V_S nearly in phase, making the circuit much more resistive than inductive.



This means that in a practical circuit, where the inductor must possess some resistance, the angle θ by which I_L lags V_S is not the 90° difference that would be expected of a pure inductor, but will be somewhere between 0° and 90°, depending on the frequency of the supply. At frequencies where X_L is much greater than R the circuit is predominantly inductive but at comparatively low frequencies where the normally small value of R may become comparable or even greater than X_L the circuit becomes more predominantly resistive.

Fig 10.2.4a The complete LCR parallel circuit.

Returning to the whole LCR circuit, three phasors, I_C, I_L and the reference phasor V_S are used to show the operation of the complete parallel circuit shown in Fig 10.2.4a.

Current phasors for L and C are used because V_L (combined with its internal resistance R_L) and V_C will be the same as they are connected in parallel across the supply. It is the currents through L and through C that will differ. The phasor for I_C leads V_S (which is also the voltage across C and L) by 90° and I_L lags V_S by somewhere between 0° and 90°, depending on component values and supply frequency.

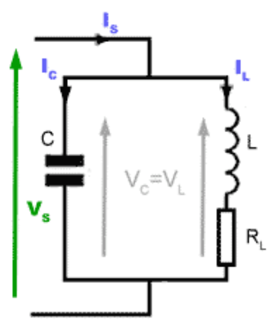
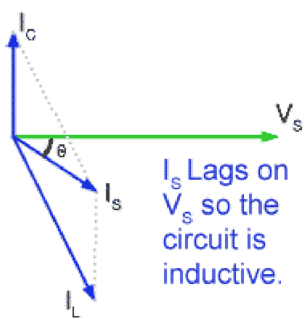


Fig 10.2.4b Phasors for the complete LCR parallel circuit.

Returning to the whole LCR circuit, three phasors, I_C, I_L and the reference phasor V_S are used to show the operation of the parallel circuit in Fig 10.2.4a.

The phasor for I_C leads V_S (which is also the voltage across C) by 90° and I_L lags V_S by somewhere between 0° and 90°, depending on component values and supply frequency.

A fourth phasor I_S (the supply current) will be the phasor sum of I_C and I_L , which in this diagram is larger than I_C. The two current phasors I_C and I_L are not in exact anti phase so the phasor for I_S is lagging that for V_S. Therefore the circuit is inductive.



Module 10.3 Parallel Resonance

Just as in series resonant circuits, there are three basic conditions in a parallel circuit. Dependent on frequency and component values, the circuit will be operating below, above or at resonance. This section describes these three conditions using phasor diagrams involving the current phasors I_C, I_L and their phasor sum I_S, with the reference phasor V_S. Note that V_R is not shown, but its presence in the circuit is indicated by the variable angle of I_L as described in Module 10.2.

Below Resonance

Firstly, if the supply frequency is low, below the resonant frequency f_r then the condition shown in Fig 10.3.1 exists, and the current I_L through L will be large (due to its comparatively low reactance). At the same time the current I_C through C will be comparatively small. Because I_C is smaller than I_L the phase angle θ will be small. Including I_S in the diagram shows that it will be lagging on V_S and therefore the circuit will appear to be INDUCTIVE. (Note that this is the opposite state of affairs to the series circuit, which is capacitive below resonance).

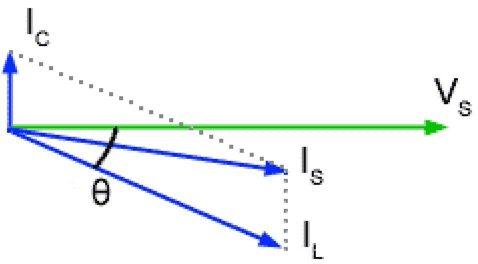


Fig. 10.3.1 Below Resonance

Above Resonance

Fig 10.3.2 shows what happens at frequencies above resonance. Here the current I_C through C will be greater than the current I_L through L, because the frequency is higher and X_C is smaller than X_L, θ is greater than in Fig 10.3.1. This gives us the condition where I_S (the phasor sum of I_C and I_L) is leading V_S and so the circuit is capacitive.

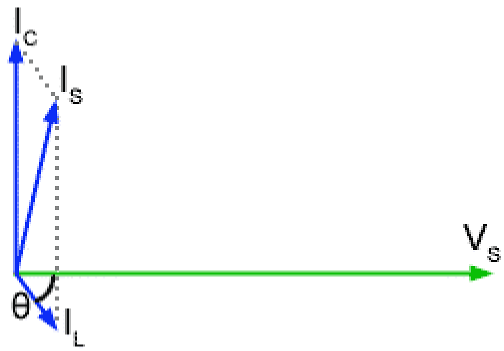


Fig 10.3.2 Above Resonance

At Resonance

At resonance the ideal circuit described in Module 10.1 has infinite impedance, but this is not quite the case in practical parallel circuits, although very nearly. Fig 10.3.3 shows the conditions for resonance in a practical parallel LCR circuit. I_C is leading V_S by 90° but I_L is not quite in anti phase (due to the resistance in the circuit´s inductive branch). **In the parallel circuit therefore, resonance must be defined as the frequency where the values of I_C and I_L are such that I_S is IN PHASE with V_S.**

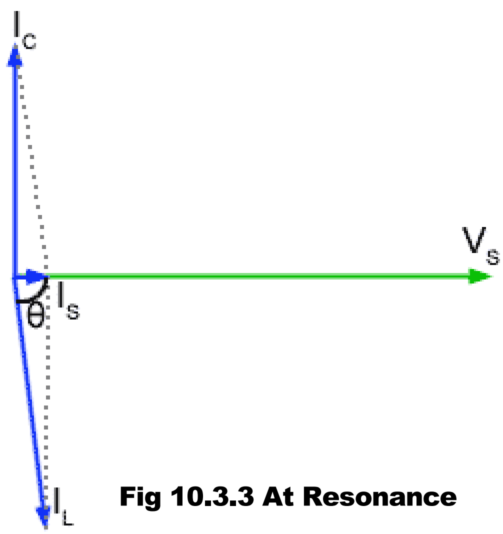


Fig 10.3.3 At Resonance

Dynamic Resistance

At resonance, Fig 10.3.3 shows that I_S is very small, much smaller than either I_C or I_L so the impedance across the parallel circuit must be very high at f_r and as I_S is in phase with V_S, the circuit impedance is purely resistive. This pure resistance that occurs only at f_r is called the DYNAMIC RESISTANCE (R_D) of the circuit and it can be calculated (in ohms) for any parallel circuit from just the component values used, using the formula:

$$R_D = \frac{L}{CR}$$

Where R is the total resistance of the circuit, including the internal resistance of L.

Current Magnification

The other important point shown in Fig 10.3.3 is the size of the phasor for I_S compared with I_C and I_L. The supply current is much smaller than either of the currents in the L or C branches of the circuit. This must mean that more current is flowing within the circuit than is actually being supplied to it!

This condition is real and is known as CURRENT MAGNIFICATION. Just as voltage magnification took place in series circuits, so the parallel LCR circuit will magnify current. The MAGNIFICATION FACTOR (Q) of a parallel circuit can be found using the same formula as for series circuits, namely;

$$Q = \frac{X_L}{R} \text{ or } \frac{2\pi f_r L}{R}$$

Adjusting for resonance.

The formula for the resonant frequency of a LCR parallel circuit also uses the same formula for f_r as in a series circuit, that is;

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

It should be noted that this formula ignores the effect of R in slightly shifting the phase of I_L. In fact the formula gives an approximate value for f_r. However, because the internal resistance of L is usually quite small, so is its effect in shifting the resonant frequency of the circuit. For this reason, the same formula may be used for f_r in both series and parallel circuits. In those practical LC circuits designed to operate at high frequencies, and where accurate control over f_r is required, it is normal for either L or C to be made adjustable in value.

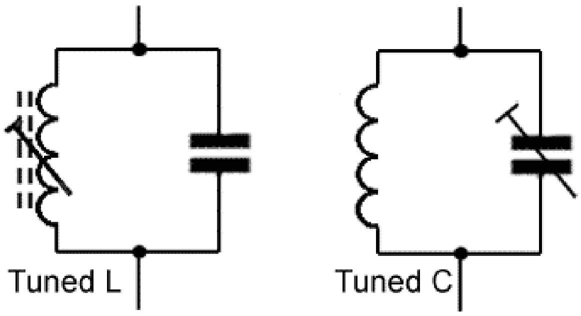


Fig 10.3.4 Parallel LC Tuned Circuits.

Notice that the usual formula for resonant frequency f_r does not have any reference to resistance (R). Although any circuit containing L must contain at least some resistance, the presence of a small amount of resistance in the circuit does not greatly affect the **frequency** at which the circuit resonates. Resonant circuits designed for high frequencies are more affected by stray magnetic fields, inductance and capacitance in their nearby environment than the very small effects of R, so most high frequency LC resonant circuits will have both screening to isolate them from external effects as much as possible, and be made adjustable over a small range of frequency so they can be accurately adjusted after assembly in the circuit.

However, although this formula is widely used at radio frequencies for both series and parallel resonance, at low frequencies where large inductors, having considerable internal resistance are used, the formula below can be used for f_r in low frequency (large internal resistance) parallel resonance calculations.

$$f_r = \frac{1}{2\pi}\sqrt{\left(\frac{1}{LC} - \frac{R^2}{L^2}\right)}$$

The need for careful adjustment after circuit assembly is often a deciding factor for the discontinued use of LC circuits in many applications. They have been widely replaced by solid-state ceramic filters and resonating crystal tuned circuits that need no adjustment. Sometimes however, there may be a problem of multiple resonant frequencies at harmonics (multiples) of the required frequency with solid state filters. A single adjustable LC tuned circuit may then also be included to overcome the problem.

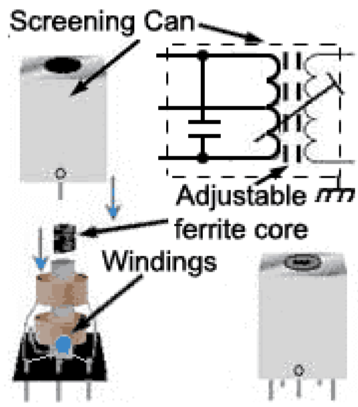


Fig 10.3.5 Tuned Transformer in a screening can

The final values for L and C would be achieved by adjusting one of the two components as shown in Fig. 10.3.4 which would be of a variable type, once the system containing the LC circuit was operating. By this method, not only is the effect of R compensated for, but also any stray inductance or capacitance in the circuit that may also affect the final value of f_r. Because, at high frequencies, magnetic fields easily radiate from one component in a circuit to another, LC tuned circuits would also be shielded (screened) by containing them in a metal screening can as shown in Fig 10.3.5.

6 Things you need to know about LCR Parallel Circuits.

(and that are different to the Series Circuit.)

- 1. **AT RESONANCE (f_r)** V_C is not necessarily exactly equal to V_L but V_S and I_S are IN PHASE
- 2.; **AT RESONANCE (f_r)** Impedance (Z) is at maximum and is called the Dynamic Resistance (R_D)
- 3. **AT RESONANCE (f_r)** Circuit current (I_S) is at a minimum.
- 4. **AT RESONANCE (f_r)** The circuit is entirely resistive.
- 5. **BELOW RESONANCE (f_r)** The circuit is inductive.
- 6. **ABOVE RESONANCE (f_r)** The circuit is capacitive.

Module 10.4 Damping

The Effects of Resistance in LC Parallel Circuits

Ignoring resistance, the resonant frequency of a LC parallel circuit is given by the same formula as is used for LC series circuits:

$$f_r = \frac{1}{(2\pi\sqrt{LC})}$$

Although this formula is only approximate due to the resistance in a LC parallel circuit, inaccuracies will be small at high frequencies. In practice we can still use the above formula for both series and parallel LC circuits.

Resistance in a parallel circuit does however substantially change the graph of impedance (Z) against frequency (f).

The graph of impedance against frequency in Fig 10.4.1 shows that, as frequency increases from zero towards resonance (f_r) the impedance of the circuit increases to a maximum value (R_D) at resonance and then decreases again for frequencies above resonance. The graph shows the FREQUENCY RESPONSE of the circuit.

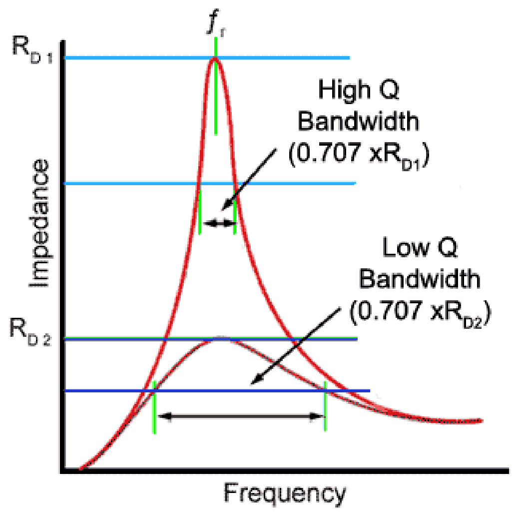


Fig 10.4.1 Parallel LCR Circuit Response Curve.

The shape of the response curve can be changed considerably by adding resistance either to the inductive branch of the circuit, e.g. increasing the internal resistance of the inductor, or by adding an external resistor called a SHUNT resistor, across the LC circuit as illustrated in Fig 10.4.2. Adding resistance by either method is called DAMPING.

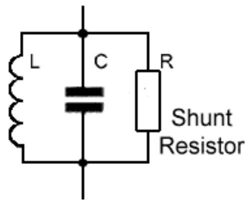


Fig 10.4.2 Damping Using a Shunt Resistor

Damping is frequently used in LC circuits to obtain a flatter response curve giving a wider bandwidth to the circuit, as shown by the lower curve in Fig 10.4.1. Applying damping has two major effects.

1. It reduces current magnification by reducing the Q factor. (R is bigger compared with XL).

2. It increases the BANDWIDTH of the circuit.

The bandwidth of a LC parallel circuit is a range of frequencies, either side of R_D, within which the total circuit impedance is greater than 0.707 of R_D.

The lower curve in Fig 10.4.1 indicates the condition where the Q factor is reduced by including a damping resistor. The Dynamic Resistance is lower (R_{D2}) and now the area above the (green) 0.707xR_{D2} line covers a wider band of frequencies.

Bandwidth, resonant frequency and Q factor in a parallel circuit are connected by the formula:

$$Q = \frac{f_r}{B} \quad \text{or} \quad B = \frac{f_r}{Q}$$

Where B is the bandwidth (upper frequency limit − lower frequency limit) in Hz.

It can be seen from these equations that, if Q is reduced while f_r is constant then bandwidth (B) must increase.

In a parallel circuit the amount of damping is set by both the value of the internal resistance of L and the value of the shunt resistor. The Q factor will be reduced by increasing the value of the internal resistance of L, The larger the internal resistance of the inductor, the lower the Q factor.

The shunt resistor has an opposite effect on Q, and the lower the value of R, the more the Q factor is reduced. If the value of the shunt resistor is halved, then so is the Q factor but the bandwidth is doubled.

Having two quite different formulae complicates the issue, but often in practice, either the internal resistance or the shunt resistance is by far the dominant effect, to the extent that the other can be ignored.

Module 10.5 LCR Parallel Quiz

What you should know.

After studying Module 10, you should:

Be able to recognise LCR parallel circuits and describe their action using phasor diagrams and appropriate equations.

Be able to describe LCR parallel Circuits at resonance and the conditions for parallel resonance.

Be able to carry out calculations on LCR parallel circuits, involving reactance, impedance, component and circuit voltages and current.

Be able to describe current magnification, dynamic impedance and Q factor, and be able to calculate their values in LCR Parallel Circuits.

Try our quiz, based on the information you can find in Module 10. Submit your answers and see how many you get right, but don't be disappointed if you get answers wrong. Just follow the hints to find the right answer and learn more about LCR parallel Circuits and Resonance as you go.

1.

With reference to Fig 10.5.1 the resonant frequency of the circuits L1/C1 and L3/C3 will be:

a) The same as L2/C2

b) Twice the frequency if L2/C2

c) Half the frequency of L2 /C2

d) Above and below the frequency of L2/C2

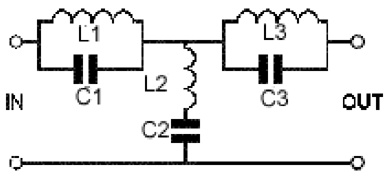


Fig 10.5.1

2.

In a parallel resonant circuit at resonance, the impedance is referred to as:

a) The equal reactance point and is at maximum.

b) The dynamic resistance and is at minimum.

c) The equal reactance point and is at minimum.

d) The dynamic resistance and is at maximum.

3.

What will be the approximate resonant frequency of a parallel LCR circuit containing L=1mH, C=1nF, R= 15Ω ?

a) 159kHz b) 251kHz c) 2.5MHz d) 2.7kHz

Continued.

4.

Phasor diagrams a-d show a single parallel resonant circuit operating at different frequencies. Which diagram shows the highest frequency?

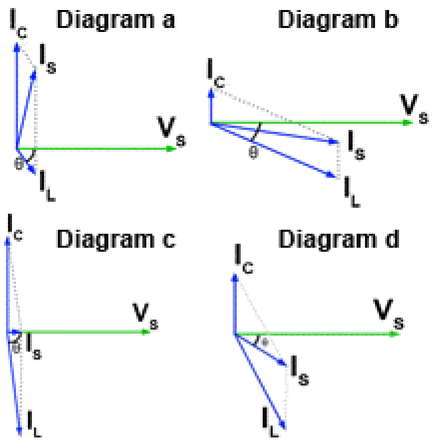
a) Diagram a

b) Diagram b

c) Diagram c

d) Diagram d

Fig 10.5.2



5.

For a parallel LCR circuit at resonance, which of the following statements is true?

a) At resonance, XC and XL are equal.

b) The phasors IL and IC are in anti−phase.

c) The current flowing into the circuit at resonance is at maximum.

d) The impedance at resonance is at maximum.

6.

If the Q factor of a parallel resonant circuit is halved, what will be the effect on the bandwidth?

a) It will be halved.

b) It will not be changed.

c) It will double.

d) It will increase by four times.

7.

Which formula in Fig 10.5.3 is correct for calculating the Q factor of a parallel resonant circuit?

a) Formula a

b) Formula b

c) Formula c

d) Formula d

Fig 10.5.3

a: $Q = 2\pi f LR$

b: $Q = \frac{1}{2\pi CR}$

c: $Q = \frac{2\pi f_r L}{R}$

d: $Q = \frac{L}{CR}$

Continued.

8.

What words are missing from the following statement? The supply current phasor for IS in a parallel LCR phasor diagram at resonance will be _____________ amplitude.

a) Lagging IC by 90°and at its Maximum.

b) In phase with VS and at its Maximum

c) In phase with VS

d) Leading IL by 90°and at its Minimum.and at its Minimum.

9.

With reference to Fig 10.5.4 What quantity is represented by axis A?

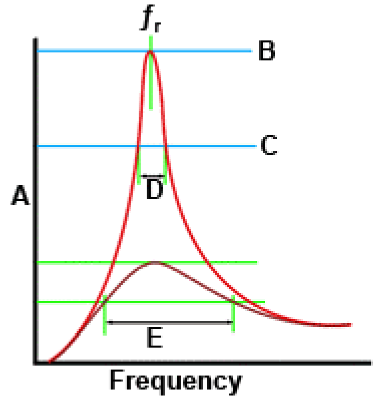
a) Inductive Reactance.

b) Current.

c) Capacitive Reactance.

d) Impedance

Fig 10.5.4



10.

What is the approximate Q factor of the circuit mentioned in Question 3?

a) 67

b) 81

c) 15

d) 32

